# Working with Confidence

*This resource was written by Colin Andersen, Mike Kolodzinski, Jan Roderick, Tony Sears and Derek Smith with the support of CASIO New Zealand. It may be freely distributed but remains the intellectual property of the authors and CASIO.*

## INTRODUCTION

This resource relates to Statistics Level 8 of Mathematics in the New Zealand Curriculum (MiNZC) document, specifically:

**Evaluate and explain the meaning of confidence intervals in estimating population parameters and in using samples for quality control;**

It demonstrates how confidence intervals can be obtained using the Casio CFX-9850G or the Casio FX-9750G graphics calculators.

**Relevant Background Information**

A confidence interval is the interval in which a certain percentage of the intervals will contain the true parameter (mean).

The formula for finding the confidence interval when the sample mean and standard deviation are given is:

$$\bar{x} - z^{-1}_{\alpha/2} \cdot \frac{s}{\sqrt{n}} < m < \bar{x} + z^{-1}_{\alpha/2} \cdot \frac{\alpha}{\sqrt{n}}$$

The notation $z^{-1}_{\alpha/2}$ is an inverse normal value found from the normal distribution tables. If α = 90%, then $z^{-1}_{\alpha/2}$ = 1.645. If α = 95%, $z^{-1}_{\alpha/2}$ = 1.96.
If α = 99%, $z^{-1}_{\alpha/2}$ = 2.58.

The formula for finding the confidence interval when the population proportion (p or π) and the sample size (n) is given by:

$$\bar{p} - z^{-1}_{\alpha/2}\sqrt{\frac{\bar{p}\bar{q}}{n}} < p < \bar{p} + z^{-1}_{\alpha/2}\sqrt{\frac{\bar{p}\bar{q}}{n}} \text{ where } \bar{q} = 1 - \bar{p}.$$

The calculations required can be performed in RUN mode.

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## PROBLEMS

*This problem is similar to a question from Bursary Mathematics with Statistics 1997.*

Problem One

A study of the occupancy rate on a cruise ship over a season of 40 sailings yielded the following results:

| | |
|---|---|
| Sample size | 40 |
| Sample mean | 0.635 |
| Sample standard deviation | 0.093 |

1. Determine a 95% confidence interval for the mean occupancy rate for this cruise ship.

2. What effect would increasing the level of confidence have on the width of this confidence interval?

Problem Two

*This problem is similar to a question from Bursary Mathematics with Statistics 1998.*

85 of the 472 seventh formers from a certain secondary school said that they smoked at least one cigarette a day. Using this result find a 90% confidence interval for the proportion of all NZ seventh form students who would say that they smoke at least one cigarette a day.

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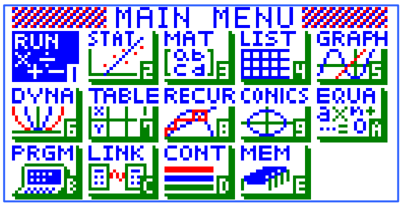
## WORKED SOLUTION

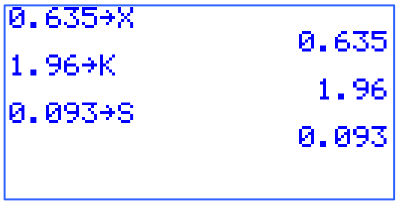



Use RUN mode to find the confidence intervals.

Problem One

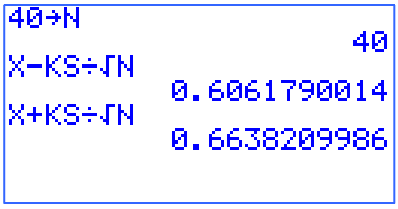
1. Determine a 95% confidence interval for the mean occupancy rate for this cruise ship.

Step One: Assign the values for the mean, z-score, standard deviation and sample size to memory locations:




0.635 → X  (We think X looks most like x.)
1.96 → K  (K is the z-score for a 95% confidence interval.)
0.93 → S  (S being the standard deviation.)
40 → N  (N being sample size.)

We like assigning the values to memory locations because we can then easily change the values.

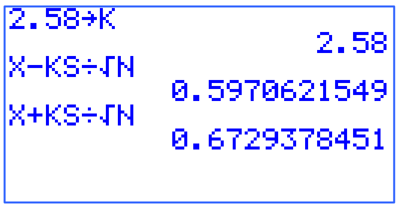



Step Two: Perform the following calculation: X-KS÷√N
(The calculator understands order of operations)

Step Three: Perform the following calculation: X+KS÷√N
(This second calculation can be found from the first by using the left arrow to edit the command line.)

From these results the 95% confidence interval is $(0.6062 \le \mu \le 0.6638)_{4sf.}$

2. What effect would increasing the level of confidence have on the width of this confidence interval?



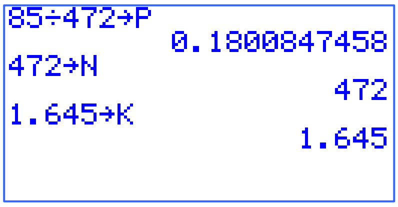

Step One: Try reassigning K to 2.58, i.e. increasing the confidence interval to 99%.

(X-KS÷√N≤ μ ≤ X+KS÷√N) now gives us $(0.5971 \le \mu \le 0.6729)_{4sf.}$

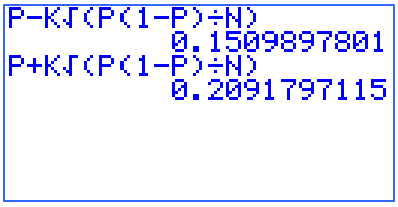
This suggests that increasing the level of confidence increases the width of the interval required.

Problem Two

85÷472 → P
472 → N
Change K to 1.645




P-K×√(P(1-P)÷N)
P+K×√(P(1-P)÷N)




So the 95% confidence interval is $(0.1510 \le p \le 0.2092)_{\text{4sf.}}$

This means that the true proportion of seventh formers who smoke at least one cigarette a day is very likely to be between 15.10% and 20.92%.

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## Further Problems to Try

1) In a random sample of 436 people 89 were found to have defective vision. Find a 95% confidence interval for the proportion of the population with defective vision.

2) To the direct question "Have you ever been given a speeding ticket?", 34 out of 100 drivers answered YES. Find a 95% confidence interval for the proportion of all drivers who have been given speeding tickets.

3) A random survey of 130 households showed that 35% had Internet access.

   a) Construct a 99% confidence interval for p, the percentage of households that have Internet access.

   b) Explain the meaning of this confidence interval.

4) When a coin is tossed 500 times, the number of heads is 217.

   a) Construct a 95% confidence interval for the long-run proportion of times that heads will appear.

   b) Is 0.5 within this confidence interval?

   c) Is the coin 'very likely' biased?

5) A company that produces a particular brand of muesli bar claims that the bars have a mean weight of 150 grams. A random sample of 150 of these muesli bars was found to have a mean weight of 153 grams and a standard deviation of 3.5 grams. Using the sample parameters as an estimate of the population parameters, find a 95% confidence interval for the average weight of this brand of muesli bar. Does this result support the company's claim?

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## Further Problems to Try - Answers

1) Between 16.63% and 24.20% of the population are likely to have defective vision.

2) Between 24.71% and 43.28% of drivers are likely to say they have been given speeding tickets.

3) a) Between 24.21% and 45.79% of households should have Internet access.

   b) We consider that 99% of similar random surveys of households would have results between the above percentages for Internet access.

4) a) The 95% confidence interval is (0.3906, 0.4774)

   b) No, 0.5 is not within this confidence interval

   c) Yes, it would appear that the coin is 'very likely' biased

5) The confidence interval is from 152.44 grams to 153.56 grams. This result does not support the company's claim that the mean weight is 150 grams. In fact the mean weight is 'very likely' more than this.